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Isometric uniqueness of a complementably universal Banach space for Schauder decomposition.

Joanna Garbulińska

Jan Kochanowski University in Kielce/Jagiellonian University in Kraków, Poland

Winter School, Hejnice, January 2013

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We present an isometric version of the complementably universal Banach space \mathbb{P} with a Schauder decomposition. The space \mathbb{P} is isomorphic to Pełczyński's space with a universal basis as well as to Kadec's complementably universal space with the bounded approximation property.

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In 1969 Pełczyński constructed a complementably universal Banach space with a Schauder basis. Two years later, Kadec constructed a complementably universal Banach space for the class of spaces with the BAP. Just after, Pełczyński showed that every Banach space with BAP is complemented in a space with a basis. Applying Pełczyński' decomposition argument, one immediately concludes that both spaces are isomorphic.

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ε -isometries			

 $(1+\varepsilon)^{-1} \cdot \|x\| \le \|f(x)\| \le (1+\varepsilon) \cdot \|x\|$

 $\forall x \in X$.

- An isometry f : X → Y that is an ε-isometry for every ε > 0,
 i.e. ||f(x)|| = ||x|| ∀x ∈ X.
- A Banach space Y is ε-complemented in X if
 Y ⊂ X
 - $T: X \to Y$ such that $||Ty y|| \le \varepsilon ||y|| \ \forall y \in Y$.

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 i.e. ||f(x)|| = ||x|| ∀x ∈ X.
- A Banach space Y is ε -complemented in X if
 - $Y \subseteq X$ • $T: X \to Y$ such that $||Ty - y|| \le \varepsilon ||y|| \quad \forall y \in Y$.

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Definition	Category theory	Main results	Bibliography
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- "0-complemented" means "complemented".
- f is a (< ε)-embedding if it is an ε'-isometric embedding for some 0 < ε' < ε.
- Y is (< ε)-complemented in X if it is ε'-complemented for some 0 < ε' < ε.
- *E* is *complementably universal* for a class of spaces if every space from the class is isomorphic to a complemented subspace of *E*.

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Definition	Category theory	Main results	Bibliography
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arepsilon -isometries			

Let X be a Banach space.

• A Schauder decomposition, (finite-dimensional decomposition) is a sequence $P_n : X \to X$ of finite rank pairwise orthogonal linear operators such that $x = \sum_{n=0}^{\infty} P_n x$ for every $x \in X$. Given such a decomposition, let $Q_n = P_0 + \cdots + P_{n-1}$. Then Q_n is a finite-rank projection $Q_n : X \to X$.

We shall say that X has k-FDD, if $k \ge \sup_{n \in \omega} ||Q_n||$. We consider 1-FDD only (called monotone FDD or monotone Schauder decomposition). Every Schauder decomposition is determined by finite-rank projections Q_n such that $Q_n Q_m = Q_{\min(n,m)}$ and $x = \lim_{n \to \infty} Q_n x$ for $x \in X$.

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Isometric uniqueness of a complementably universal Banach space for Schauder decomposition.

Definition	Category theory	Main results	Bibliography
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Initial object			

Fix $\varepsilon > 0$ and fix a surjective linear operator $f: X \to Y$ such that

 $(1+\varepsilon)^{-1}||x|| \le ||f(x)|| \le ||x||$

for $x \in X$. Consider the following category $\mathfrak{K}_{f}^{\varepsilon}$. The objects: $i: X \to Z, j: Y \to Z$ such that

- $||i|| \le 1$ and $||j|| \le 1$;
- $||i(x) j(f(x))|| \le \varepsilon ||x||$ for $x \in X$.

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Definition	Category theory	Main results	Bibliography
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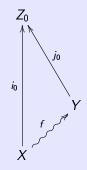
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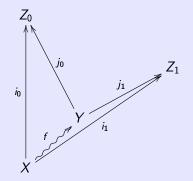


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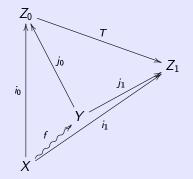


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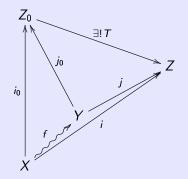


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An initial object.



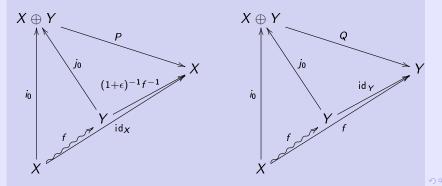
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Lemma1

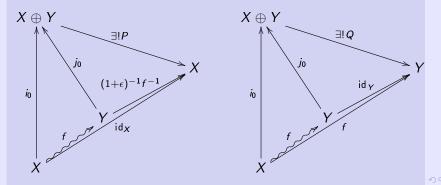
The category $\mathfrak{K}_f^{\varepsilon}$ has an initial object (i_0, j_0) such that both i_0, j_0 are canonical isometric embeddings into $X \oplus Y$ with a suitable norm $\|\cdot\|$ and there exist projections $P: X \oplus Y \to X$ and $Q: X \oplus Y \to Y$ $(\|P\| \le 1 \text{ and } \|Q\| \le 1)$.



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Initial object			

Lemma1

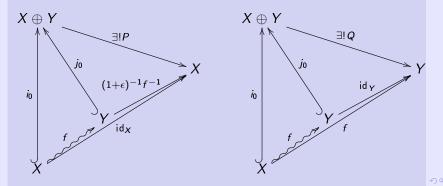
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Lemma1

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Initial object			
1) Defi	ne		
	$G = \{(x, -f(x)) \in X$	$\times Y : x \in \varepsilon^{-1}B_X\}.$	
2) Le	t <i>K</i> be the convex hull of (<i>B</i>	$B_X \times \{0\} \cup (\{0\} \times B_Y)$	$_{Y})\cup G$.
We wil	l show that the norm		
$\ (x,y)$	$\ _{K} = \inf\{\ x_0\ _{X} + \ y_1\ _{Y} + \varepsilon$	$\varepsilon \ x_2\ _X : (x,y) =$	
$(x_0, 0)$	$+(0, y_1)+(x_2, -f(x_2)), (x, y_1)$	$y) \in K$ }, is as require	d.
Define	$i_0(x) = (x, 0), j_0(y) = (0, y)$		
• Fi	rstly we show that (i_0, j_0) is	an object of $\mathfrak{K}_{f}^{\varepsilon}$:	
	• $ i_0 _K \le 1$ and $ j_0 _K \le 1$;		
	• $\ i_0(x) - j_0(f(x))\ _{\mathcal{K}} \leq \varepsilon \ x\ $	for $x \in X$;	
• VV	'e prove that i_0 and j_0 are isc	metric embeddings.	
Next st	tep is to show that (i_0, j_0) is	an initial object of $\mathfrak{K}^arepsilon_f$	
• Gi	ven an object (i,j) of $\mathfrak{K}^arepsilon_f$, de	efine	
	T(x,y) =	i(x)+j(y).	

• We show that $||T||_{\mathcal{K}} \leq 1$.

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Initial object			

1) Define

$$G = \{(x, -f(x)) \in X \times Y : x \in \varepsilon^{-1}B_X\}.$$

• Firstly we show that (i_0, j_0) is an object of $\mathfrak{R}_f^{\varepsilon}$. • $||i_0||_K < 1$ and $||i_0||_K < 1$; • $||i_0(x) - i_0(f(x))||_{\mathcal{K}} < \varepsilon ||x||$ for $x \in X$; • We prove that i_0 and j_0 are isometric embeddings. • Given an object (i, j) of $\Re_{\epsilon}^{\varepsilon}$, define

• We show that $||T||_{K} \leq 1$.

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Initial object			
1) Defir	e		
	$G = \{(x, -f(x)) \in X$	$\times Y : x \in \varepsilon^{-1}B_X\}.$	
-	K be the convex hull of (E	$B_X \times \{0\} \cup (\{0\} \times B_Y)$	⁄)∪G.
	show that the norm		
	$_{K} = \inf\{\ x_{0}\ _{X} + \ y_{1}\ _{Y} + \ y_{1$		
	$+(0, y_1) + (x_2, -f(x_2)), (x,$		d.
Define <i>i</i>	$j_0(x) = (x, 0), \ j_0(y) = (0, y)$		
	stly we show that (i_0, j_0) is	an object of $\mathfrak{K}_f^arepsilon$:	
	• $\ i_0\ _K \le 1$ and $\ j_0\ _K \le 1;$		
(• $\ i_0(x) - j_0(f(x))\ _{\mathcal{K}} \le \varepsilon \ x\ $	for $x \in X$;	
• We	prove that i_0 and j_0 are iso	ometric embeddings.	
Next ste	ep is to show that (i_0, j_0) is	an initial object of $\mathfrak{K}^{\varepsilon}_{f}$	
• Giv	ven an object (i,j) of $\mathfrak{K}^arepsilon_f$, de	efine	
	T(x,y) =	i(x) + j(y).	

• We show that $||T||_{\mathcal{K}} \leq 1$.

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Initial object			
1) Define			
	$G = \{(x, -f(x)) \in X\}$	$X \times Y : x \in \varepsilon^{-1}B_X\}.$	
2) Let <i>P</i>	<pre>K be the convex hull of (</pre>	$B_X \times \{0\}) \cup (\{0\} \times E$	$(B_Y) \cup G$.
-	now that the norm		
$\ (x,y)\ _{\mathcal{K}}$	$= \inf\{\ x_0\ _X + \ y_1\ _Y +$	$\varepsilon \ x_2\ _X : (x, y) =$	
$(x_0, 0) + ($	$(0, y_1) + (x_2, -f(x_2)), (x_1)$	$(y) \in K$, is as require	ed.
Define i ₀ ($f(x) = (x, 0), \ j_0(y) = (0, y)$		
	ly we show that (i_0, j_0) is	an object of $\mathfrak{K}_{f}^{\varepsilon}$:	
	$ i_0 _K \le 1 \text{ and } j_0 _K \le 1;$		
	$\ i_0(x)-j_0(f(x))\ _{\mathcal{K}}\leq \varepsilon\ x$		
	prove that i_0 and j_0 are is		
Next step	is to show that (i_0, j_0) is	s an initial object of R	ε f
• Giver	n an object (i,j) of $\mathfrak{K}_{f}^{arepsilon}$, c	define	

T(x,y) = i(x) + j(y).

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• We show that $||T||_{K} \leq 1$.

Definition 000	Category theory ○○○○●○○○○	Main results 000	Bibliography ○
Initial object			
1) Define			
	$G = \{(x, -f(x)) \in X\}$	$Y \times Y : x \in \varepsilon^{-1}B_X\}.$	
2) Let <i>K</i>	be the convex hull of ($B_X \times \{0\} \cup (\{0\} \times B_Y)$	$(Y) \cup G$.
We will sho	ow that the norm		
	$= \inf\{\ x_0\ _X + \ y_1\ _Y +$		
$(x_0, 0) + (0)$	$(y_1) + (x_2, -f(x_2)), (x_2)$	$,y)\in K\}$, is as require	d.
Define i ₀ (x	$) = (x, 0), j_0(y) = (0, y)$	() <u>.</u>	
 Firstly 	we show that (i_0, j_0) is	an object of $\mathfrak{K}^{arepsilon}_{f}$:	
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- $||i_0||_{\mathcal{K}} \le 1$ and $||j_0||_{\mathcal{K}} \le 1$;
- $\|i_0(x) j_0(f(x))\|_{\mathcal{K}} \le \varepsilon \|x\|$ for $x \in X$;

• We prove that i_0 and j_0 are isometric embeddings.

Next step is to show that (i_0, j_0) is an initial object of \Re_f^{ε} . • Given an object (i, j) of \Re_f^{ε} , define

T(x,y) = i(x) + j(y).

• We show that $||T||_{\mathcal{K}} \leq 1$.

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1) Define			
	$G = \{(x, -f(x)) \in X\}$	$X \times Y : x \in \varepsilon^{-1}B_X\}.$	

2) Let K be the convex hull of $(B_X imes \{0\}) \cup (\{0\} imes B_Y) \cup G$. We will show that the norm

 $\begin{aligned} \|(x,y)\|_{\mathcal{K}} &= \inf\{\|x_0\|_{\mathcal{X}} + \|y_1\|_{Y} + \varepsilon \|x_2\|_{\mathcal{X}} : (x,y) = \\ (x_0,0) + (0,y_1) + (x_2, -f(x_2)), (x,y) \in \mathcal{K}\}, \text{ is as required.} \\ \text{Define } i_0(x) &= (x,0), \ j_0(y) = (0,y). \end{aligned}$

• Firstly we show that (i_0, j_0) is an object of $\mathfrak{R}_f^{\varepsilon}$:

•
$$||i_0||_{\mathcal{K}} \leq 1$$
 and $||j_0||_{\mathcal{K}} \leq 1$;

• $\|i_0(x) - j_0(f(x))\|_{\mathcal{K}} \le \varepsilon \|x\|$ for $x \in X$;

• We prove that i_0 and j_0 are isometric embeddings.

Next step is to show that (i_0, j_0) is an initial object of $\mathfrak{K}_f^{\varepsilon}$.

• Given an object (i, j) of $\mathfrak{K}_{f}^{\varepsilon}$, define

$$T(x,y)=i(x)+j(y).$$

• We show that $||T||_{\mathcal{K}} \leq 1$.

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Initial object			

3) Define linear operators $P: X \oplus Y \to X$ and $Q: X \oplus Y \to Y$ as:

• $P(x, y) = x + (1 + \varepsilon)^{-1} f^{-1}(y)$

•
$$Q(x, y) = f(x) + y$$

4) We check that $\|P\|_X \leq 1$ and $\|Q\|_Y \leq 1$, so these operators are projections.

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3) Define linear operators $P: X \oplus Y \to X$ and $Q: X \oplus Y \to Y$ as:

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$$P(x, y) = x + (1 + \varepsilon)^{-1} f^{-1}(y)$$

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Let \mathfrak{K} be a category. A *Fraïssé sequence* in \mathfrak{K} is an inductive sequence \vec{U} satisfying the following conditions: (U) For every $A \in \mathfrak{K}$ there exists $n \in \mathbb{N}$ such that $\mathfrak{K}(A, U_n) \neq \emptyset$;

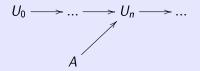
$$U_0 \longrightarrow \dots \longrightarrow U_n \longrightarrow \dots$$

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(A) For every $n \in \mathbb{N}$ and for every morphism $f \in \mathfrak{K}(U_n, B)$, where $B \in \mathfrak{K}$, there exist $m \in \mathbb{N}$, m > n and $g \in \mathfrak{K}(B, U_m)$ such that $u_n^m = g \circ f$.

$$U_0 \longrightarrow U_1 \longrightarrow \dots U_n \longrightarrow \dots U_k \longrightarrow U_m \dots$$

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We now define the relevant category \mathfrak{K} . The objects of \mathfrak{K} are rational finite-dimensional Banach spaces.

 $X_0 \qquad X_1 \qquad \dots \qquad X_n \qquad \dots$

Given rational finite-dimensional spaces E, F, an \Re -arrow is a pair (e, P) of rational linear operators $e : E \to F$, $P : F \to E$ such that:

(P1) e is a rational isometric embedding. (P2) $P \circ e = id_E$ and $||P|| \le 1$, where E is the domain of e. Now we use the fact that every countable category with amalgamations has a Fraïssé sequence.

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$$X_0 \xrightarrow[\stackrel{e_0^1}{\longrightarrow} X_1 \xrightarrow[\stackrel{e_1^2}{\longrightarrow} \dots \xrightarrow[\stackrel{e_{n-1}^n}{\xrightarrow} Y_n]} X_n \xrightarrow[\stackrel{e_n^{n+1}}{\xrightarrow} \dots$$

Given rational finite-dimensional spaces E, F, an \Re -arrow is a pair (e, P) of rational linear operators $e : E \to F$, $P : F \to E$ such that:

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Let us consider the following extension property of a Banach space X:

(E) Given a pair $E \subseteq F$ of finite-dimensional Banach spaces such that E is complemented in F, given an isometric embedding $i: E \to X$ such that i[E] is complemented in X, for every $\varepsilon > 0$ there exists an ε -isometric embedding $g: F \to X$ such that $||g| \upharpoonright E - i|| < \varepsilon$ and g[F] is ε -complemented in X.

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Theorem (Uniqueness)

Let \mathbb{P} and \mathbb{K} be Banach spaces satisfying condition (E) and let $h: A \to B$ be a bijective linear isometry between complemented finite-dimensional subspaces of \mathbb{P} and \mathbb{K} , respectively. Then for every $\varepsilon > 0$ there exists a bijective linear isometry $H: \mathbb{P} \to \mathbb{K}$ that is ε -close to h. In particular, \mathbb{P} and \mathbb{K} are linearly isometric.



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Theorem (Universality)

Let X be a Banach space with a monotone FDD. Then there exists an isometric embedding $e: X \to \mathbb{P}$ such that e[X] is 1-complemented in \mathbb{P} .

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